

() , GPS Galileo, BeiDou, -
(QZSS, IRNSS), -
(, EGNOS, WAAS, MSAS) -
40-50 ().
Galileo , GPS
,
200 [1-3].
800 [1].
(,
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,
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(100 Javad,
NovAtel [1, 2]).
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« « « »», « (.-). ».

[4, 30].

[6].

WiFi. (A-GNSS),

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-
-
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»., «. «. «. «. «. «. «. -1» [5].

[7, 8].

(RTK-).

[9–11].

[12],

$$x_k = \Phi_k x_k + w_k \quad (1)$$

$$z_k = H_k x_k + \Lambda N + v_k, \quad (2)$$

$\Phi_k, H_k, \Lambda =$

$w_k, v_k =$

$; N = m-$

,

(1)

x_k

(1)

x_k

(2)

H_k

Λ

N

(2)

$$, \quad [9-11, 29].$$

(float) N

$$(fixed) \quad , \quad x \quad N.$$

$\mathbb{Z}^m,$

$$N \quad x \quad ,$$

N^*

$$N^* = \arg \min_{N \in \mathbb{Z}^m} \underbrace{(N - \hat{N})^T P_N^{-1} (N - \hat{N})}_q, \quad (3)$$

$$\hat{N}, P_N -$$

$$x,$$

$$\zeta = |x| + \varepsilon, \quad \varepsilon - \sigma_\varepsilon^2, \quad N^* [11]:$$

$$N^* = \arg \min_{N \in \mathbb{Z}^m} \underbrace{[(N - \hat{N})^T P_N^{-1} (N - \hat{N}) + (\zeta - |\overset{\vee}{x}(N)|)^2 / \sigma_\zeta^2]}_q, \quad (4)$$

$$\overset{\vee}{x}(N) - x, \quad (2)$$

$$N, \quad \sigma_\zeta^2 = \overset{\vee}{x}(N)^T P_{x/N} \overset{\vee}{x}(N) / |\overset{\vee}{x}(N)|^2 + \sigma_\varepsilon^2 -$$

$$z \quad N, \quad P_{x/N} -$$

$$\overset{\vee}{x}(N).$$

$$\hat{N} \quad P_N \quad \text{LAMBDA-} \quad (3), (4).$$

$$[12], \quad P_N, \quad ,$$

$$(3), (4) \quad N^*$$

$$N \quad (3), (4) \quad q$$

$$q_j = q_{j-1} + (N_j - \hat{N}_{j/j-1})^2 / \sigma_{j/j-1}^2, \quad j = \overline{1, m}, \quad q_0 = 0, \quad (5)$$

$$\widehat{N}_{1/0} = \widehat{N}_1, \quad \sigma_{1/0}^2 = \sigma_1^2; \quad \widehat{N}_{j/j-1}, \quad \sigma_{j/j-1}^2, \quad j = \overline{2, m} -$$

$$\bar{N}_{j-1} = (N_1, \dots, N_{j-1})^T$$

$$N_j, \quad \quad \quad N \quad P_N . \quad \quad \quad (3) \quad q = q_m ,$$

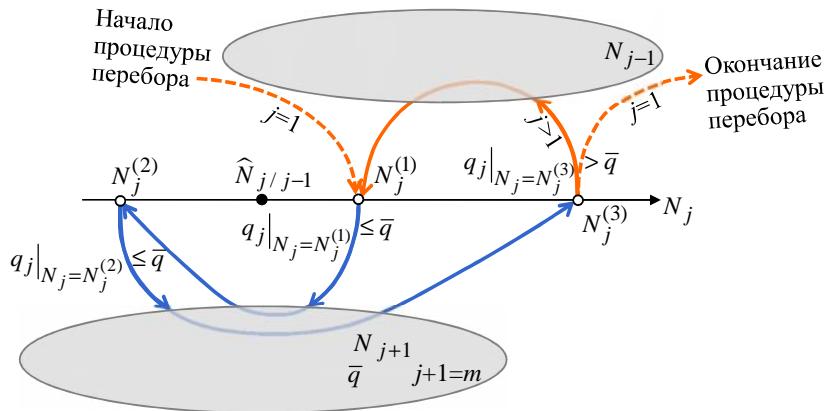
$$\vee \quad \quad \quad q, \quad \quad \quad \quad \quad \quad (4), \quad \quad \quad (5)$$

$$q = q_m + (\zeta - |x(N)|)^\zeta / \sigma_\zeta^\zeta, \quad , \quad (3), (4)$$

$$q_N \quad . \quad \overline{q} \; . \quad , \quad \overline{q}$$

[11] ,

N.



$$\ldots 1. \qquad \qquad \qquad N_j \ (j < m)$$

$$N \qquad \qquad N_1. \qquad \qquad j$$

$$j=1, \quad N \quad , \quad . \quad . \quad . \quad 1 \quad ,$$

$$\frac{j}{N}, \dots$$

$$N_j^{(s_j)} = \begin{cases} \langle \widehat{N}_{j/j-1} \rangle, & s_j = 1; \\ N_j^{(s_j-1)} + (-1)^{s_j} (s_j - 1), & s_j > 1 \quad (N_j^{(1)} - \widehat{N}_{j/j-1}) < 0; \\ N_j^{(s_j-1)} + (-1)^{s_j-1} (s_j - 1), & s_j > 1 \quad (N_j^{(1)} - \widehat{N}_{j/j-1}) \geq 0, \end{cases} \quad (6)$$

$$\begin{aligned}
s_j = 1, 2, \dots & \quad N_j, \langle \hat{\mathbf{I}} \rangle - \\
q_j & \quad s_j \quad N_j, \\
N_j^{(1)}, \dots & \quad - N_j^{(2)}, N_j^{(3)} \quad \dots, \\
q_j|_{N_j=N_j^{(s_j)}} \leq \bar{q} & \quad , \\
N_j, & \quad \\
(6) & \quad
\end{aligned}$$

$$\begin{aligned}
& \quad , \\
& \quad \bar{q} \cdot \\
& \quad j \\
\widehat{N}_{j/j-1} & \quad N, \\
N, & \quad q \leq \bar{q}, \quad \bar{q} \\
q, & \quad , \\
& \quad , \quad (6), \\
N^*, & \quad ,
\end{aligned}$$

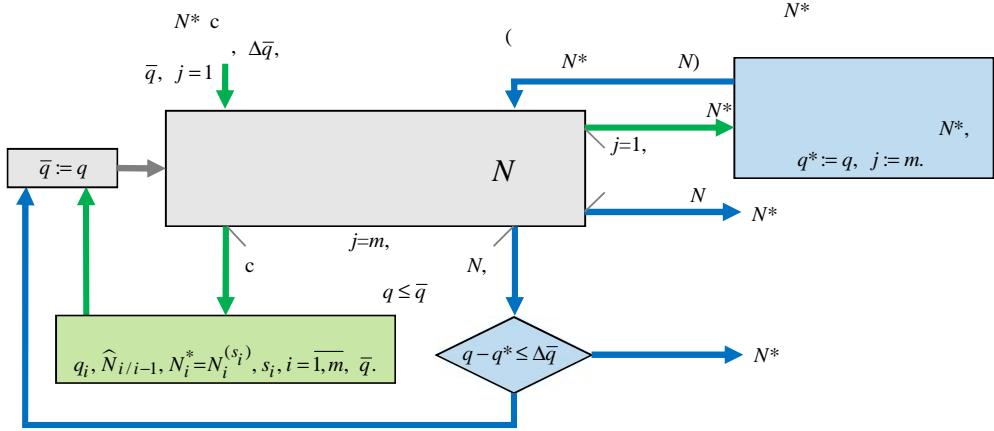
$$\begin{aligned}
& \quad , \\
& \quad (\bar{z}_k)) \\
g^* = \mathbf{P}(N = N^* | \bar{z}_k) & \quad , \\
g^{**}. & \quad g^* / g^{**} \quad g^* \\
g^* / g^{**} & \quad , \quad N^* \\
& \quad x. \\
& \quad , \\
& \quad - \quad N^*, \quad - \quad g^* \\
g^* / g^{**}. & \quad , \\
& \quad , \quad g^* \quad g^* / g^{**}. \\
& \quad , \\
g^* / g^{**} & \quad . 2.
\end{aligned}$$

$$\begin{aligned}
\Delta \bar{q} > 0 & \quad \gamma^{\min} \quad g^* / g^{**} \quad \Delta \bar{q} = 2 \ln \gamma^{\min}. \\
g^* & \quad N^* \\
, & \quad (\bar{q}, \quad N^*), \\
& \quad N,
\end{aligned}$$

$$q \leq \bar{q} , \quad \Sigma := \Sigma + e^{-q/2} \quad \Sigma = e^{-q^{*}/2} .$$

$$e^{-q^{*}/2} / \Sigma \leq \bar{g}^{*} , \quad \bar{g}^{*} -$$

N^{*}.



. 2.

N^* ,

$$N^* - \quad , \quad q \leq \bar{q} ,$$

$$N.$$

(bootstrapping) [29] $\bar{q} = q(N = N^b)$

$$N^*, N^b$$

$$- \quad \quad j = \overline{1, m} \quad -$$

N*

$$N^b \\ N^* \\ N^*, \\ , \\ N^* \\ N,$$

(. . . 2).

g^* ,

N*

,

$$\bar{q} = q(N=N^B)$$

« -1 » [5],
[7, 8].

[13]. (),

[14-
17]

$$x_k = \Phi_k x_{k-1} + w_k + B_k D(\gamma) \eta, \quad z_k = H_k x_k + C_k D(\gamma) \eta + v_k. \quad (7)$$

, z_k

$$p^i = \mathbf{P}(\gamma^j = \delta_j^i, j = \overline{1, N}), \quad \delta_j^i = \begin{cases} 1, & - \\ 0, & + \end{cases}, \quad , \quad x_0, \\ w_k, v_k, \eta \quad \gamma.$$

$$\eta \quad \quad \quad . \quad \quad \quad w_k, \ v_k$$

$$g_k^i = \mathbf{P}(\gamma^j = \delta_j^i, j = \overline{1, N} \mid \bar{z}_k), \quad \bar{z}_k,$$

$$[14-17], \quad (7) \quad x_k \\ \eta^i, \quad , \quad , \quad \dots \\ \gamma^j = \delta_j^i, \quad j = \overline{1, N}.$$

$$\begin{aligned} & \bar{x}_k = (x_k^T, \eta^T)^T \\ & , \quad \dots, \gamma^j = 1 \\ & j = \overline{1, N} \quad , \quad D(\gamma) = E. \\ & (\text{. . . (7)}) \quad \gamma^j = \delta_j^i, \quad j = \overline{1, N} \quad \gamma^j = 1, \quad j = \overline{1, N}, \quad \tilde{\eta}^i = 0, \quad \tilde{\eta}^i = \dots, \\ & \eta \quad \eta^i, \end{aligned}$$

$$g_k^i \propto \sqrt{\prod_{j=1, N, j \neq i} \sigma_{\eta_j}^2 / |P_{\tilde{\eta}_k^i}^*|} \exp\left(-\frac{1}{2}(\hat{\eta}_k^{i*})^T (P_{\tilde{\eta}_k^i}^*)^{-1} \hat{\eta}_k^{i*}\right) p^i, \quad (8)$$

$$\sigma_{\eta^j}^2 = \eta^j; \hat{\tilde{\eta}}_k^{i*}, P_{\hat{\tilde{\eta}}_k^i}^* = \tilde{\eta}^i$$

$$(8) \quad P_{\tilde{r}_i^i}^*$$

$$i \in \overline{1, N}, \quad N^4$$

$$\left(\hat{P}_\eta \right)^* = \left| P_\eta^* \right|, \quad [13].$$

$$\left(P_{\mathfrak{N}^i}^* \right)^{-1}$$

$$\bar{\eta}^i = \left((\tilde{\eta}^i)^T, \eta^i \right)^T ,$$

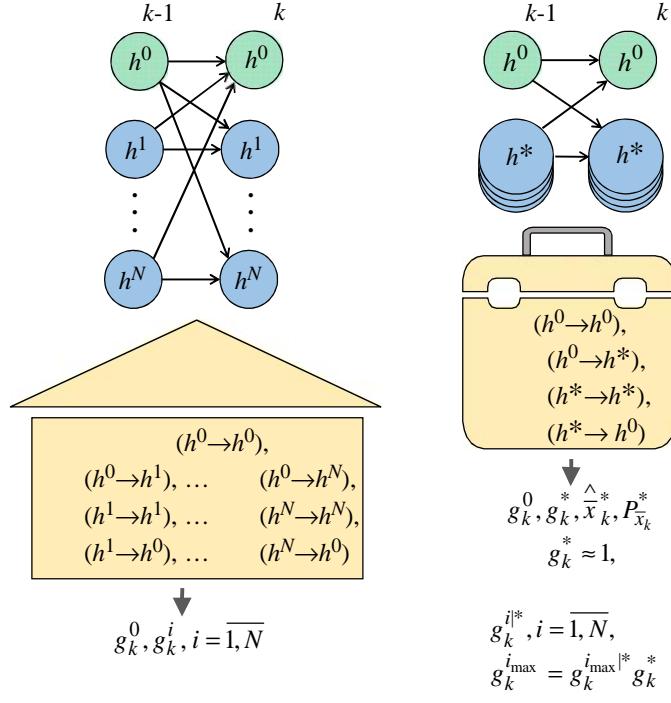
$$(8) \quad \begin{aligned} & \left(P_{\eta}^* \right)^{-1} \\ & \left(P_{\tilde{\eta}^i}^* \right)^{-1} \\ & \vdots \\ & \left| P_{\tilde{\eta}^i}^* \right| \end{aligned} \quad .$$

$$\begin{aligned} \Theta_{\tilde{\eta}^i} &= \left(P_{\tilde{\eta}^i}^* \right)^{-1}, \quad \Theta_{\tilde{\eta}^i \eta^i} \\ &= \left(P_{\tilde{\eta}^i}^* \right)^{-1}, \quad , \quad (9) \\ \left(P_{\tilde{\eta}^i}^* \right)^{-1} &= \Theta_{\tilde{\eta}^i, \eta^i}^T / \Theta_{\eta^i}, \\ &\quad \left| P_{\tilde{\eta}^i}^* \right| \\ (9) & \quad g^i \end{aligned}$$

$$, \quad : \quad N \quad , \quad (n+1)- \\ n+N. \\ , \quad n, \quad . \\ , \quad n \\ ,$$

$$N \quad - \quad h^0: \gamma = 0, \\ N \quad - \quad h^i: \gamma^j = \delta_j^i, j = \overline{1, N}, \\ h^0 \rightarrow h^i, i = \overline{1, N}, \\ , \quad 3N+1$$

$$g_k^0, g_k^i, i = \overline{1, N}, \quad \chi_k^i = (x_k^T, \eta^i)^T \\ i = \overline{1, N}.$$



$$\text{. 3.} \quad (\quad) \quad (\quad)$$

$$\begin{aligned}
& , \\
h^0 & , h^*. \quad h^* \\
& , \\
g_k^* \approx 1 & \quad (8) \quad h^* \\
i = i_{\max} & \quad g_k^{i_{\max}} = \mathbf{P}(\gamma^j = \delta_j^i, j = \overline{1, N} \mid \bar{z}_k, h^*). \\
g_k^i = \mathbf{P}(\gamma^j = \delta_j^i, j = \overline{1, N} \mid \bar{z}_k) & = g_k^{i_{\max}} g_k^*, \quad - \\
\hat{\chi}_k^i & \quad P_{\chi_k^i}. \\
& \quad h^{1*}, h^{2*} \dots
\end{aligned}$$

$$\begin{aligned}
& i- \\
(8) \quad \tilde{\eta}^i & \quad \gamma^j, \quad 1. \quad - \\
, \quad j & \quad , \quad \gamma^j, \quad 1,
\end{aligned}$$

$$\sigma_{\eta^j}^2 \quad j \quad 1 \quad N,$$

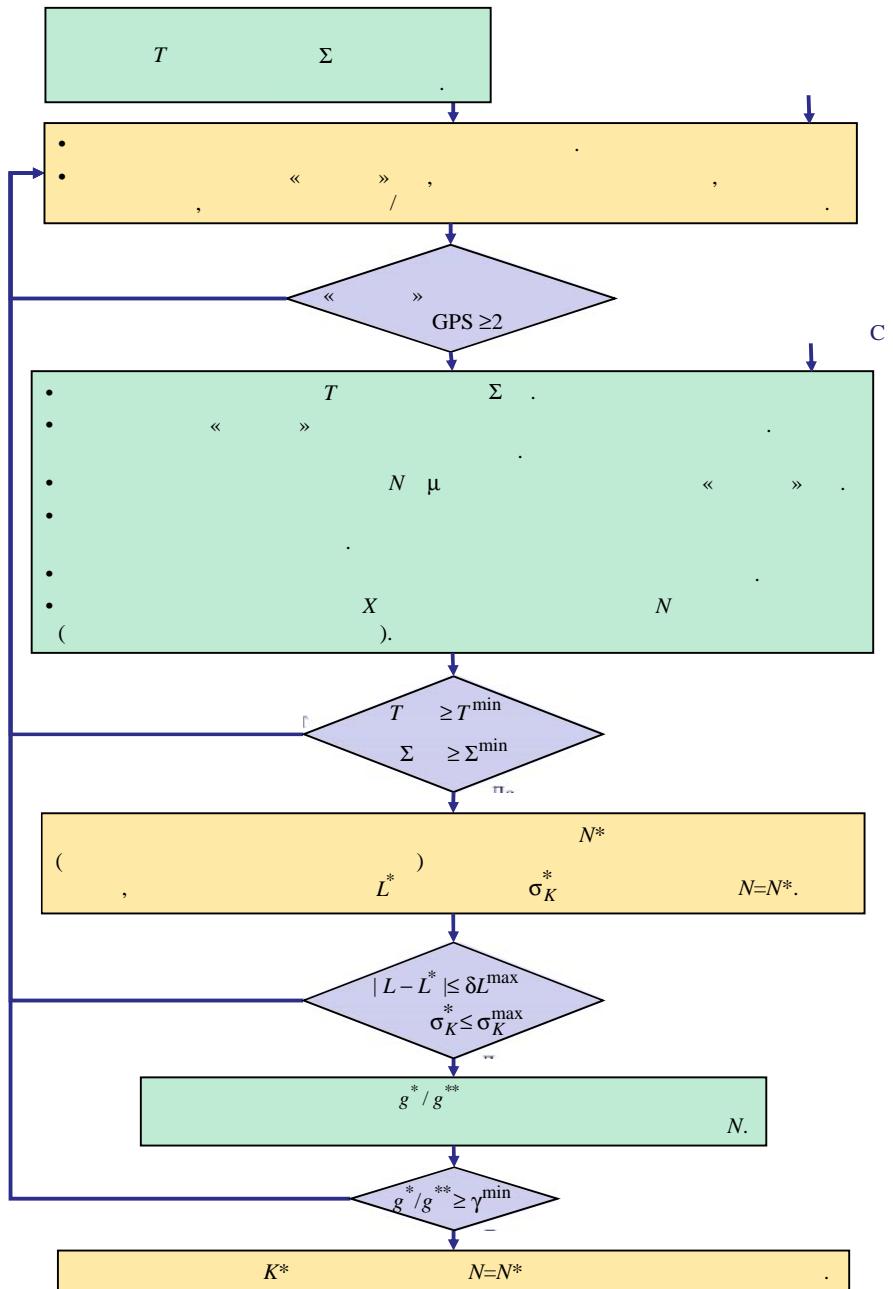
$\gamma^j = 1.$

η^i

[13].

« « « » »
[18, 19].

4



4. , , T^{\min} , Σ^{\min} , δL^{\max} ,

$$\sigma_K^{\max}, \gamma^{\min} -$$

, « », GPS ≥ 2», ,

$$() \quad \sigma_K^* \\ 3 \sigma_K^{\max}.$$

$$T^*, \Sigma^*, \gamma$$

$$\sigma_K^* \leq \sigma_K^{\max}, \quad 3\sigma_K^{\max}.$$

$$\sigma_K^{\max}$$

$$).$$

$$(2) \quad Y = H S + \Lambda N + \mu + \nu.$$

$$X = (S^T, N^T, \mu^T)^T, \quad S - , \mu -$$

$$(1).$$

$$N^*, K^*, N = N^*.$$

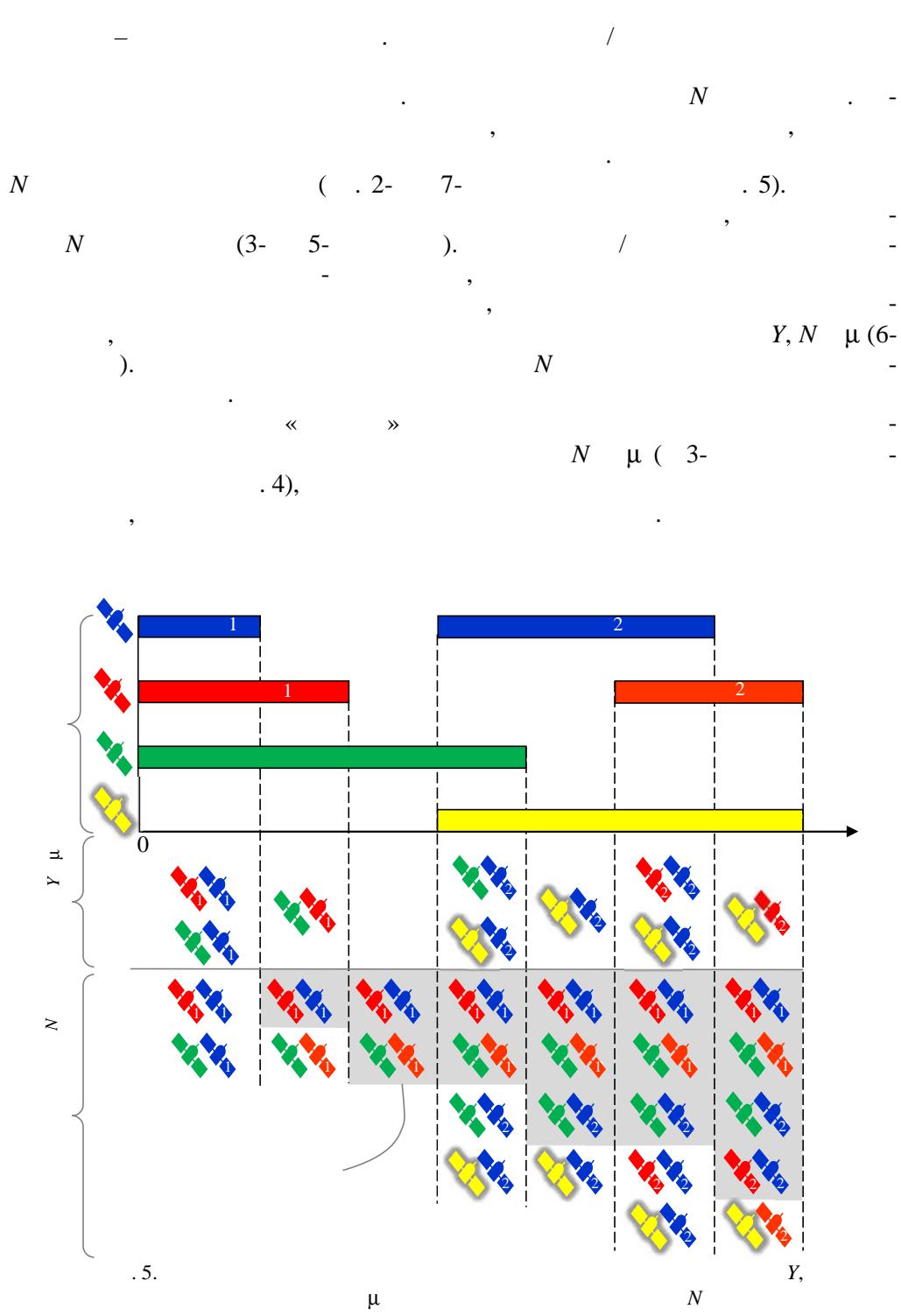
$$,$$

$$, \ll$$

$$\gg ,$$

$$5 \quad Y, \mu \quad N$$

$$,$$

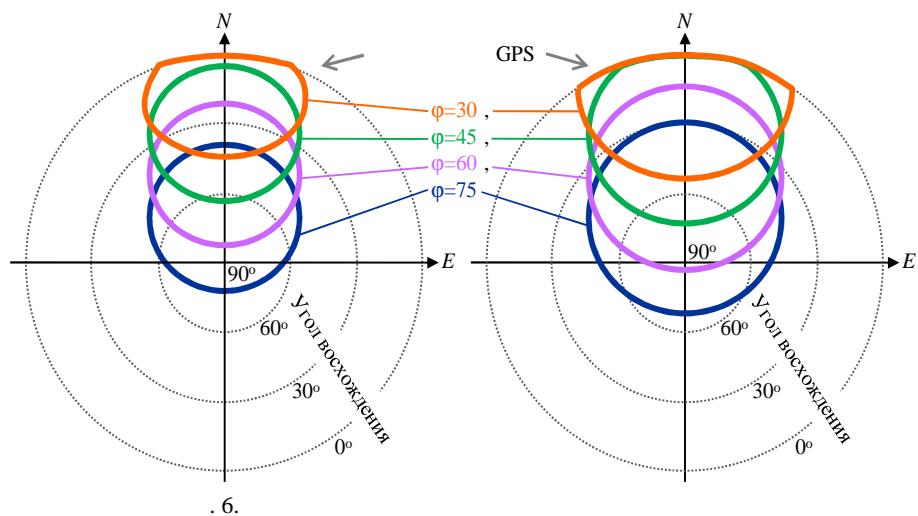


40-70°.

6

GPS

GPS



GPS

1,5

0,2°

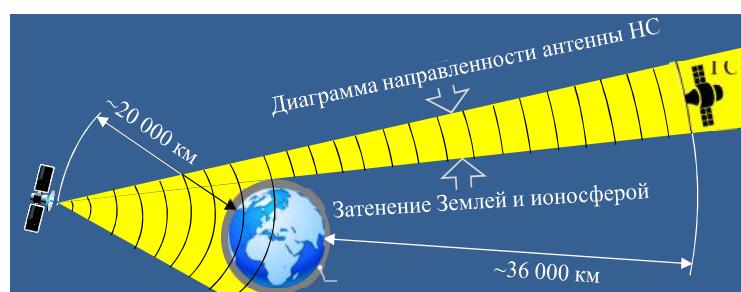
180, 120 90°

		0,2°,	
	, °		
	180	1,1	1,7
	120	1,5	2,5
	90	2	5
	180	1	1
	120	1,5	2,5
	90	2	4

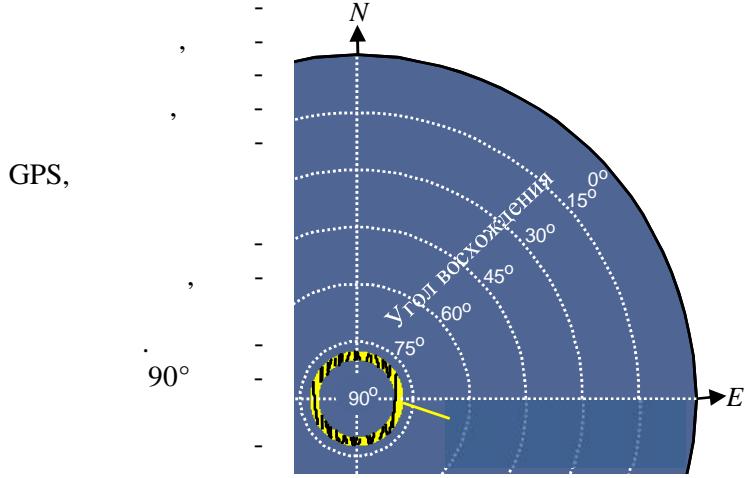
(,), ,), ,

, , , [20, 21].

GPS-
()
,
(. 7).



. 7.



24

[21–23].

[24].

r^{CRS} v^{CRS}
CRS (Celestial Reference
System) [22, 25]:

$$\dot{r}^{\text{CRS}} = v^{\text{CRS}}, \quad \dot{v}^{\text{CRS}} = -GM_{\oplus} r^{\text{CRS}} / |r^{\text{CRS}}|^3 + F(r^{\text{CRS}}, v^{\text{CRS}}, t). \quad (10)$$

GM_{\oplus} — ; $F(\cdot)$

— , t — . $F(\cdot)$

$F(\cdot)$

\mathbb{C} ,

t_0

— \tilde{r}_0, \tilde{v}_0 ; $P_{\tilde{r}_0}, P_{v_0}$.

\tilde{C}_0

$$\sigma_{\mathbb{C}_0}^2. \quad (10)$$

$$t_k = t_0 + k\Delta t, \quad k = 1, 2, \dots \quad (\quad \Delta t > 0 \quad - \\ \delta t_k^{\text{GPS}}, \quad -$$

$$\delta t_{\max}^{\text{GPS}} \ll \Delta t .$$

$$n_k \quad \quad \quad t_k, \dots \quad \quad \quad [t_k; t_k + \delta t_{\max}^{\text{GPS}}], \quad -$$

$$\begin{aligned} \mathcal{Z}_{\mathcal{D}_k}^+ &= \mathcal{D}_k^+ (r(t_k + \sigma t_k^-), t_k + \sigma t_k^-) + v_k^+ + \sigma t_k^+ + \zeta_{\mathcal{D}_k}, \\ \mathcal{Z}_{\mathcal{D}_k}^i &= \dot{\mathcal{Z}}_k^i (r(t_k - \sigma t_k^+, \mathbf{s}_{\text{GPS}}), r(t_k, \mathbf{s}_{\text{GPS}}), r(t_k + \sigma t_k^+, \mathbf{s}_{\text{GPS}}), \mathbf{s}_{\text{GPS}}) + \zeta_k^i. \end{aligned} \quad (1.1)$$

$$\mathcal{Z}_{\tilde{\mathcal{D}}_k}^j = \tilde{D}_k^j \left(r(t_k + \delta t_k^{\text{cris}}), v(t_k + \delta t_k^{\text{cris}}), t_k + \delta t_k^{\text{cris}} \right) + \delta f_k + \zeta_{\tilde{\mathcal{D}}_k}^j, \quad (11)$$

$$j=1, n_k \quad - \quad ; \quad \mathcal{D}_k^J(\cdot),$$

$$\mathcal{D}_k^J(\cdot) \longrightarrow \quad , \quad -$$

$$J^- \quad \ll \qquad \qquad \gg \quad -$$

$\frac{r}{v}$ $\frac{v}{r}$

-84: δt_L , δf_L =

$$\dots, \varepsilon_i, \varepsilon_j, \dots$$

$$, \varsigma_{\mathcal{D}_k}, \varsigma_{\tilde{\mathcal{D}}_k} \quad , \quad v_k$$

$$-\zeta_{\mathcal{D}_k}^j, \zeta_{\tilde{\mathcal{D}}_k}^j,$$

$$\vartheta_k^j - \dots$$

$$: \hspace{1cm} (10), \hspace{1cm} r, v \hspace{1cm} -$$

$$t_k = t_0 + k\Delta t, \quad k=1, 2, \dots$$

($n_k = 1$).

(11),
 $\frac{s_1 - s_2}{s_1 + s_2}$,

$$\left(\quad \quad \quad \right) \qquad \qquad \qquad n_l \qquad \quad .$$

$$m_k = n_k - 1$$

[10, 26].

$$\Delta r, \Delta v, \Delta \vartheta$$

$$r, v, (10),$$

$$\begin{matrix} k \\ (10), \\ k-1 & r, v, \\ \Delta r, \Delta v. \end{matrix}$$

[27],

$$\Delta r, \Delta v, \tilde{r}_0, \tilde{v}_0, (10)$$

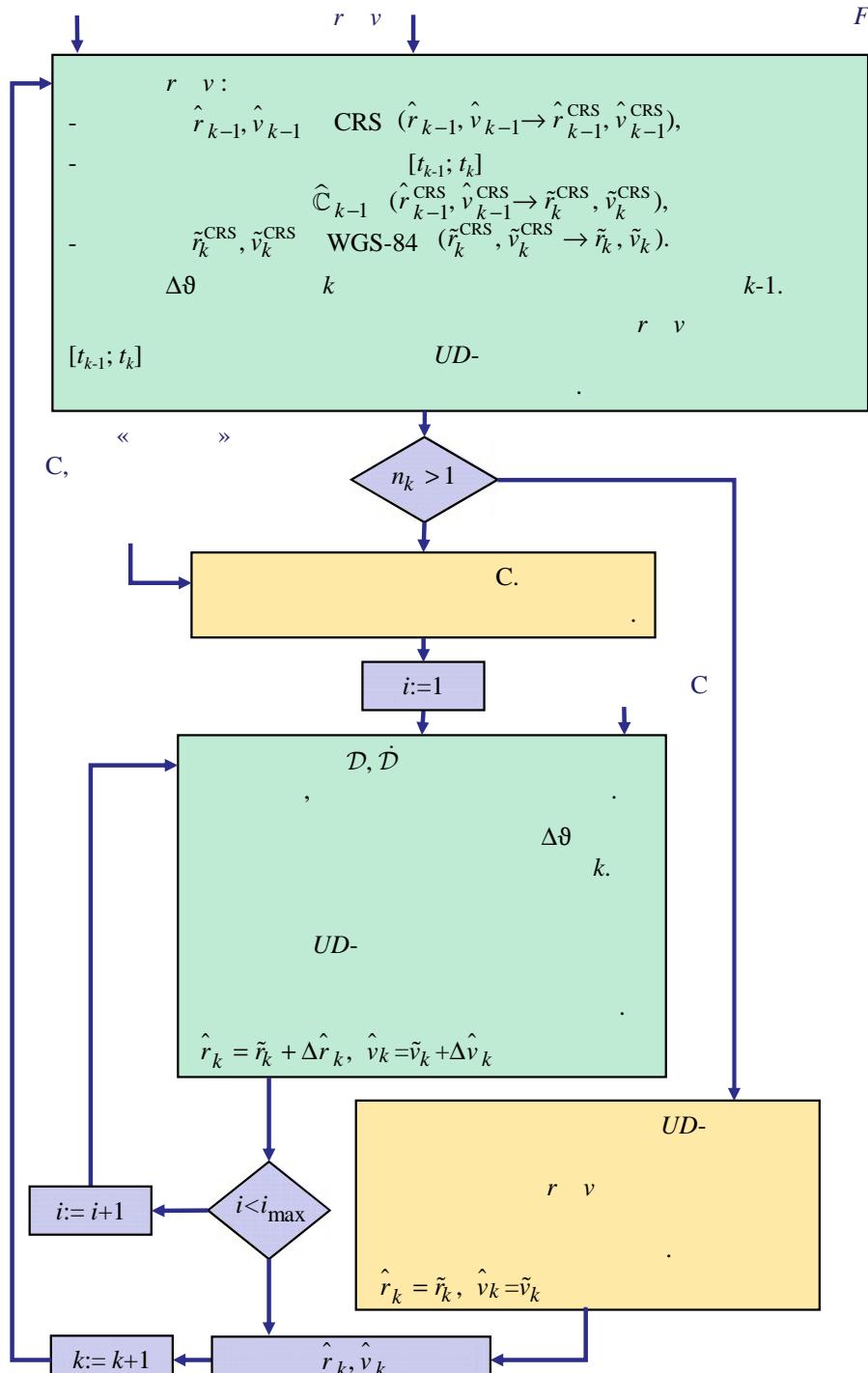
$$\begin{matrix} UD- \\ P \\ U \end{matrix} [28].$$

$$P = UDU^T, \quad \begin{matrix} D, \\ UD- \\ P \end{matrix}$$

. 9.

WGS-84 CRS
IERS – International Earth Rotation Service (., [25]).

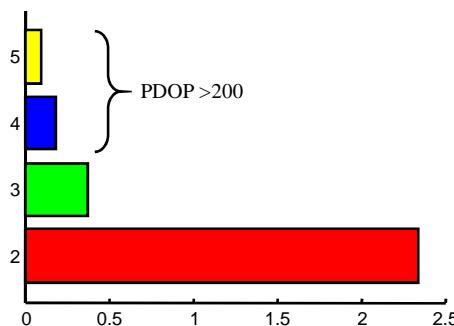
TRS – Terrestrial Reference System.



. 9.

$$X_k = (\Delta r_k^T, \Delta v_k^T, \mathbb{C}, \Delta \vartheta_k^T)^T,$$

$$\Delta r_k, \Delta v_k - \tilde{r}_k, \tilde{v}_k, \hat{r}_k, \hat{v}_k,$$



24-

. 10

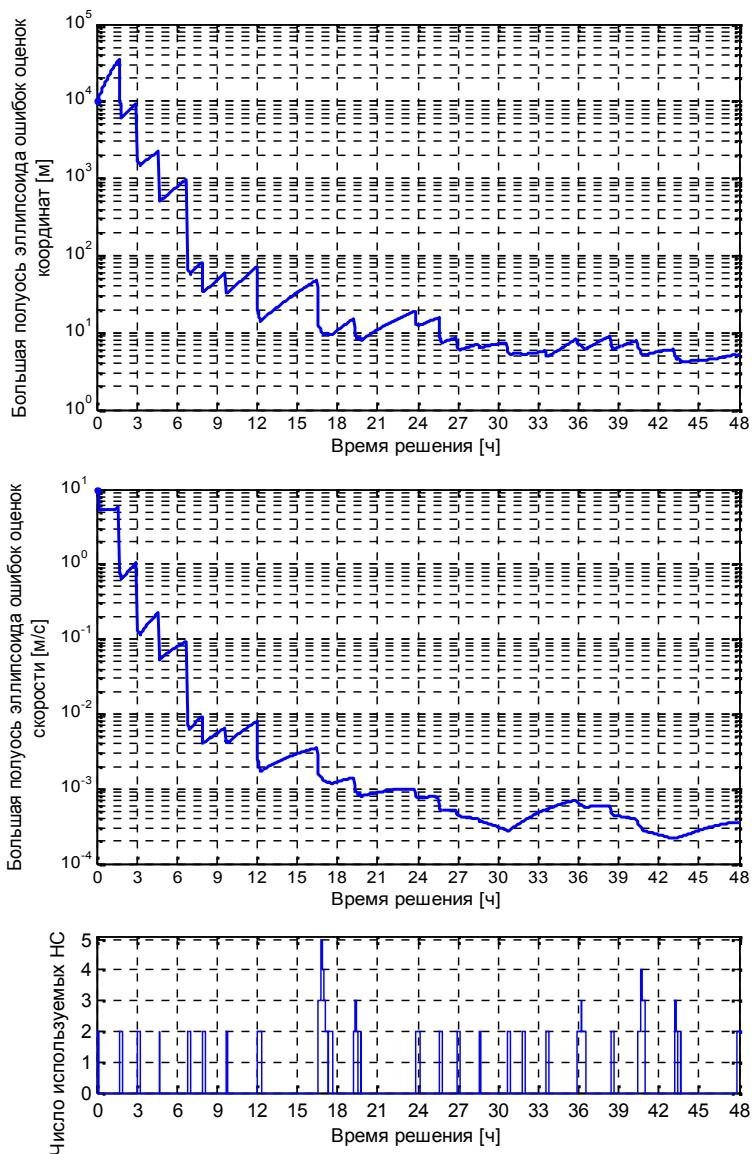
GPS - 2 5 ()

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0-17

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11.

(14-29-00160).

Abstract. The paper focuses on four problems of GNSS measurement processing. The difficulty of problems lies in large volume of available data and limited resources of onboard equipment (phase measurement ambiguity resolution, fault detection and isolation); or in insufficient volume and fragmentarity of data (multiantenna heading determination in constrained conditions, navigation of geostationary satellites). Theoretical foundations of proposed solutions are given. Experimental results are provided.

Key words: GNSS measurement processing, experimental results.